

Fig. 14. Measured and calculated frequency response of the experimental bandpass filter.

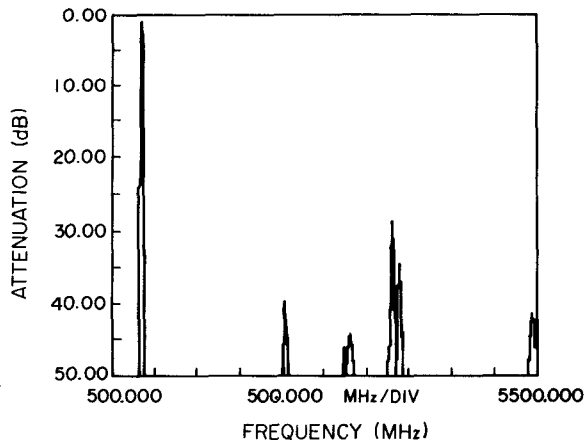


Fig. 15. Measured spurious response of the fabricated bandpass filter.

filter, it was necessary to tune resonators with tuning screws, but no adjustment of coupling capacitors was required, so coupling capacitors were precisely determined by photolithographic technology.

Fig. 13 shows the outer view of the fabricated filter. Its physical dimensions are 80 mm (length) \times 14 mm (width) \times 20 mm (height) and its volume is 22.4 cm³.

Fig. 14 shows the measured and calculated frequency response of the experimental bandpass filter. The solid and dotted lines indicate the measured and calculated responses, respectively. The fabricated filter performance shows close coincidence with the design results. Therefore, the propriety of the design formulas is experimentally verified. Passband insertion loss was obtained at 1.6 dB at midband and 1.85 dB at band edge.

Fig. 15 shows the measured spurious response. It is similar to the spurious response of the computer simulation. But the spurious response near 3.8 GHz doesn't appear in the simulation. The spurious response near 3.8 GHz is different from that of the TEM mode in a coaxial resonator, UIR, and SIR. Consideration concerning spurious frequency and the diameter of the resonator proved this unknown mode to be a TE₂₁ mode.

VI. CONCLUSIONS

A method of designing capacitively coupled bandpass filters with arbitrarily structured resonators was established and the fabricated filter performance showed close coincidence with the design results. It is shown that wide stopband characteristics can

be realized by combining quarter-wavelength uniform impedance resonators (UIR's) with stepped impedance resonators (SIR's). The special feature of this filter is that the spurious response can be controlled by the impedance ratio K of the SIR and the combination of UIR's and SIR's.

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REFERENCES

- [1] M. Makimoto and S. Yamashita, "Compact bandpass filters using stepped impedance resonators," *Proc. IEEE*, vol. 67, pp. 16-19, Jan. 1979.
- [2] M. Makimoto and S. Yamashita, "Bandpass filters using parallel coupled strip-line stepped impedance resonators," in *IEEE MTT-S Int. Microwave Symp.*, May 1980, pp. 141-143.
- [3] S. Yamashita and M. Makimoto, "Miniaturized coaxial resonator partially loaded with high-dielectric-constant microwave ceramics," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-31, pp. 697-703, Sept. 1983.
- [4] G. L. Matthaei, L. Young, and E. M. T. Jones, *Microwave Filters, Impedance-Matching Networks, and Coupling Structures*. New York: McGraw-Hill, 1964.
- [5] S. Kawashima *et al.*, "Dielectric properties of Ba(Zn_{1/3}Nb_{2/3})O₃-Ba(Zn_{1/3}Ta_{2/3})O₃ ceramics at microwave frequency," in *Proc. First Meeting on Ferroelectric and Their Applications*, Apr. 1978, pp. 293-296.

Conservation Laws for Distributed Four-Ports

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Abstract—Condition of reciprocity, losslessness, bilateral symmetry, transversal symmetry, and semireciprocity is given for a four-port in terms of its impedance, admittance, scattering, and transfer representation. Corresponding conditions are also presented for the system coupling matrix of a uniform distributed circuit. The results are applied to an anisotropic stratified waveguide.

I. INTRODUCTION

Classical network analysis has provided invaluable analytical tools for the design of microwave devices. Circuit methods, however, are far less prevalent in integrated optics, acoustooptics, and related areas. The purpose of this paper is to provide, in tabular form, conditions for certain properties of integrated circuits. These properties include reciprocity, losslessness, bilateral and transversal symmetry, and semireciprocity. The last one of these, exhibited by anisotropic media, appears to be novel.

The conservation laws are expressed in terms of various, commonly used terminal representations, such as impedance, admittance, scattering, scattering transfer, etc., as well as in terms of the system coupling matrix of a uniform distributed network. Conversion from one representation to another is facilitated by a computer program developed by one of the authors (RA). The

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tabulated expressions have proved to be useful in numerical analysis and design, where they can be implemented to test for the validity and accuracy of computed results.

Since a number of the integrated devices are four-ports, or can be viewed as such, as for example the anisotropic slab waveguide supporting a TE-TM hybrid mode, we found it practical to restrict this treatment to four-ports, although the analysis has been extended to $2n$ -ports.

Network representations and the system coupling matrix of uniform distributed parameter devices are defined in Section II. Generalized Pauli matrices, introduced in Section III, make it possible to express the conservation laws in compact form. A table, listing conditions for reciprocity, losslessness, bilateral symmetry, transversal symmetry, and semireciprocity, is presented in Section IV. An application from the area of electromagnetic wave propagation in anisotropic media concludes the paper.

II. NOTATION

Referring to Fig. 1, illustrating a four-port, its terminal parameter sets and two possible axes of symmetry, the following notation is adopted: $(\mathbf{a}) = \text{col}[a_1, a_2, a_3, a_4]$ is the vector of the incident waves, and analog vectors denote the reflected waves (\mathbf{b}) , the port voltages (\mathbf{V}) and the port currents (\mathbf{I}) . These vectors are linearly related via the network matrices. Thus, $\mathbf{V} = \mathbf{Z}\mathbf{I}$, $\mathbf{I} = \mathbf{Y}\mathbf{V}$, and $\mathbf{b} = \mathbf{S}\mathbf{a}$. The impedance transfer and scattering transfer matrices are defined by

$$\begin{bmatrix} V_1 \\ V_2 \\ I_1 \\ I_2 \end{bmatrix} = \mathbf{Q} \begin{bmatrix} V_3 \\ V_4 \\ -I_3 \\ -I_4 \end{bmatrix} \quad \text{and} \quad \begin{bmatrix} a_1 \\ a_2 \\ b_1 \\ b_2 \end{bmatrix} = \mathbf{T} \begin{bmatrix} b_3 \\ b_4 \\ a_3 \\ a_4 \end{bmatrix} \quad (1)$$

respectively. Two other often used representations, denoted by \mathbf{A} and \mathbf{M} , are related to the \mathbf{T} matrix through the expressions

$$\mathbf{A} \triangleq \Pi \mathbf{T} \Pi \triangleq \tilde{\mathbf{T}} \quad \text{and} \quad \mathbf{M} = \Pi \mathbf{T}^{-1} \Pi \quad (2)$$

respectively, where

$$\Pi = \Pi^{-1} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}.$$

In addition to the terminal parameters, Fig. 1 also indicates waves traveling forward and backward along the x coordinate which is seen as the axial coordinate of a distributed network, internal to the black box. The previously defined transfer matrix \mathbf{M} can therefore also be viewed as a function of x

$$\mathbf{a}(x) = \mathbf{M}(x) \mathbf{a}(0) \quad (3)$$

where, in distinction from the previously defined terminal vector \mathbf{a}

$$\mathbf{a}(x) \triangleq \text{col}[a_1^+(x), a_1^-(x), a_2^+(x), a_2^-(x)]. \quad (4)$$

Alternatively, the voltages and currents at a given x location can be referred to those at a reference location by the expression

$$\tilde{\mathbf{g}}(x) = \mathbf{Q}^{-1}(x) \tilde{\mathbf{g}}(0) \quad (5)$$

where $\mathbf{g}(x) = \text{col}[V_1(x), I_1(x), V_2(x), I_2(x)]$ and $\tilde{\mathbf{g}}(x) = \Pi \mathbf{g}(x)$. Clearly, one must define a transformation to link voltages and currents on the one hand to forward and backward traveling waves on the other. Adopting the so-called traveling-wave representation [1], this transformation is given by

$$\mathbf{g}(x) = \Omega \mathbf{a}(x) \quad (6)$$

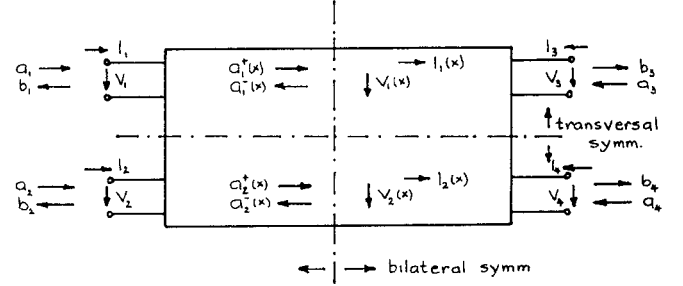


Fig. 1. To the definition of four-port representations.

where

$$\Omega = \frac{1}{\sqrt{2}} \begin{bmatrix} Z_1^{1/2} & Z_1^{1/2} & 0 & 0 \\ Z_1^{-1/2} & -Z_1^{-1/2} & 0 & 0 \\ 0 & 0 & Z_2^{1/2} & Z_2^{1/2} \\ 0 & 0 & Z_2^{-1/2} & -Z_2^{-1/2} \end{bmatrix} \quad (7)$$

and Z_1 and Z_2 are appropriately chosen characteristic impedances.

Bearing in mind that the network is distributed in the x direction, and that the wave parameters at $x + dx$ are linearly related to those at x , one can specify at the outset that

$$\frac{d}{dx} \mathbf{a}(x) = -j \mathbf{R} \mathbf{a}(x) \quad (8)$$

where \mathbf{R} is the system coupling matrix [2]. In keeping with the assumption of uniformity, \mathbf{R} must be a constant. When (3) is substituted into (8) and consideration is given to the fact that $\mathbf{a}(0)$ is arbitrary, one finds that

$$\frac{d}{dx} \mathbf{M}(x) = \mathbf{M}'(x) = -j \mathbf{R} \mathbf{M}(x). \quad (9)$$

It can also be shown [2] that \mathbf{R} and $\mathbf{M}(x)$ have the same set of eigenvectors and that, as a consequence, they commute.

III. GENERALIZED PAULI MATRICES

Generalized Pauli matrices provide the means to compactly express the conservation laws of four-ports. They are given by (\mathbf{E}_2 is the 2×2 identity matrix):

$$\sigma_0 = \mathbf{E}_4 = \begin{bmatrix} \mathbf{E}_2 & 0 \\ 0 & \mathbf{E}_2 \end{bmatrix}, \sigma_1 = \begin{bmatrix} \mathbf{E}_2 & 0 \\ 0 & -\mathbf{E}_2 \end{bmatrix} \\ \sigma_2 = \begin{bmatrix} 0 & \mathbf{E}_2 \\ \mathbf{E}_2 & 0 \end{bmatrix}, \sigma_3 = j \begin{bmatrix} 0 & -\mathbf{E}_2 \\ 0 & \mathbf{E}_2 \end{bmatrix} \quad (10)$$

and their tilde transforms $\tilde{\sigma}_i = \Pi \sigma_i \Pi$, $i = 0, 1, 2, 3$. The generalized Pauli matrices are unimodular (determinant is unity), involutive (their square is the identity), and Hermitian. Furthermore, $\sigma_k \sigma_l + \sigma_l \sigma_k = 2 \delta_{kl} \mathbf{E}_4 = \tilde{\sigma}_k \tilde{\sigma}_l + \tilde{\sigma}_l \tilde{\sigma}_k$, $\sigma_k \tilde{\sigma}_l = \tilde{\sigma}_k \sigma_l$, $\sigma_k \sigma_l = j \sigma_m$, and $\tilde{\sigma}_k \tilde{\sigma}_l = j \tilde{\sigma}_m$, $k, l, m = 1, 2, 3$ in cyclic order. The generalized Pauli matrices represent a subset of the so-called Dirac matrices [3] obtained as the Kronecker (or direct) product of the ordinary Pauli matrices.

IV. RECIPROCITY, LOSSLESSNESS, AND SYMMETRY

Five conditions will be considered: reciprocity, losslessness, bilateral symmetry, transversal symmetry, and semireciprocity. Some of these properties, and the constraints associated with them, are well known [1]. Others, such as semireciprocity observed in waveguides containing anisotropic media, have only recently been investigated [5]. Also considered novel is the extension of these conditions to the system coupling matrix \mathbf{R} .

TABLE I
SUMMARY OF THE CONSERVATION LAWS

| | Reciprocity | Losslessness | Bilat. symm. | Trans. symm. | Semireciprocity |
|---|--|--|--|---|--|
| Z | $Z = Z^T$ | $Z = -Z$ | $Z = \sigma_2 Z \sigma_2$ | $Z = \tilde{\sigma}_2 Z \tilde{\sigma}_2$ | $Z = \tilde{\sigma}_1 Z \tilde{\sigma}_1$ |
| Y | $Y = Y^T$ | $Y = -Y$ | $Y = \sigma_2 Y \sigma_2$ | $Y = \tilde{\sigma}_2 Y \tilde{\sigma}_2$ | $Y = \tilde{\sigma}_1 Y \tilde{\sigma}_1$ |
| Q | $Q^{-1} = \sigma_3 Q^T \sigma_3$ | $Q^{-1} = \sigma_2 Q \sigma_2$ | $Q^{-1} = \sigma_1 Q \sigma_1$ | $Q = \tilde{\sigma}_2 Q \tilde{\sigma}_2$ | $Q^{-1} = \tilde{\sigma}_1 \sigma_3 Q^T \sigma_3 \tilde{\sigma}_1$ |
| S | $S = S^T$ | $S^{-1} = S$ | $S = \sigma_2 S \sigma_2$ | $S = \tilde{\sigma}_2 S \tilde{\sigma}_2$ | $S = \tilde{\sigma}_1 S \tilde{\sigma}_1$ |
| T | $T^{-1} = \sigma_3 T^T \sigma_3$ | $T^{-1} = \sigma_2 T \sigma_1$ | $T^{-1} = \sigma_2 T \sigma_2$ | $T = \tilde{\sigma}_2 T \tilde{\sigma}_2$ | $T^{-1} = \tilde{\sigma}_1 \sigma_3 T^T \sigma_3 \tilde{\sigma}_1$ |
| A | $A^{-1} = \tilde{\sigma}_3 A^T \tilde{\sigma}_3$ | $A^{-1} = \tilde{\sigma}_1 A \tilde{\sigma}_1$ | $A^{-1} = \tilde{\sigma}_2 A \tilde{\sigma}_2$ | $A = \sigma_2 A \sigma_2$ | $A^{-1} = \sigma_1 \tilde{\sigma}_3 A^T \tilde{\sigma}_3 \sigma_1$ |
| M | $M^{-1} = \tilde{\sigma}_3 M^T \tilde{\sigma}_3$ | $M^{-1} = \tilde{\sigma}_1 M \tilde{\sigma}_1$ | $M^{-1} = \tilde{\sigma}_2 M \tilde{\sigma}_2$ | $M = \sigma_2 M \sigma_2$ | $M^{-1} = \sigma_1 \tilde{\sigma}_3 M^T \tilde{\sigma}_3 \sigma_1$ |
| R | $R = -\tilde{\sigma}_3 R^T \tilde{\sigma}_3$ | $R = \tilde{\sigma}_1 R \tilde{\sigma}_1$ | $R = -\tilde{\sigma}_2 R \tilde{\sigma}_2$ | $R = \sigma_2 R \sigma_2$ | $R = -\sigma_1 \tilde{\sigma}_3 R^T \tilde{\sigma}_3 \sigma_1$ |

A summary of the five conditions as they apply to the eight representations defined in Section II is presented in Table I. The entries in the first and second column are derived from the appropriate condition expressed in terms of the impedance or the scattering matrix, using the laws of transformation linking the various representations. For example, the reciprocity condition of R obtains from the reciprocity condition of $M(x)$

$$M(x) \tilde{\sigma}_3 M^T(x) \tilde{\sigma}_3 = E_4. \quad (11)$$

Taking the derivative of (11), substituting (9) and making use of (11) and the properties of the generalized Pauli matrices, we find that for a reciprocal network the system coupling matrix must satisfy the expression

$$R = -\tilde{\sigma}_3 R^T \tilde{\sigma}_3. \quad (12)$$

A similar derivation for the losslessness condition is given in [4]. The conditions for bilateral symmetry are obtained by stipulating that a representation be unchanged when the corresponding four-port is rotated around the bilateral symmetry axis, shown in Fig. 1. Thus, in the scattering transfer representation, for example, it is required that

$$\begin{bmatrix} a_3 \\ a_4 \\ b_3 \\ b_4 \end{bmatrix} = T \begin{bmatrix} b_1 \\ b_2 \\ a_1 \\ a_2 \end{bmatrix}. \quad (13)$$

Analogously, the scattering matrix of a transversally symmetric four-port must satisfy

$$\begin{bmatrix} b_2 \\ b_1 \\ b_4 \\ b_3 \end{bmatrix} = S \begin{bmatrix} a_2 \\ a_1 \\ a_4 \\ a_3 \end{bmatrix}. \quad (14)$$

A semireciprocal four-port is defined by the Z matrix representation

$$Z \triangleq \begin{bmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ -Z_{12} & Z_{22} & Z_{23} & Z_{24} \\ Z_{13} & -Z_{23} & Z_{33} & Z_{34} \\ -Z_{14} & Z_{24} & -Z_{34} & Z_{44} \end{bmatrix}. \quad (15)$$

Such a network is a series connection of two four-ports, one purely reciprocal in which there is a link between ports 1 and 3

and ports 2 and 4, but there is no coupling between line 1 and line 2, and another, purely "antireciprocal" in which no coupling exists between ports 1 and 3 and between ports 2 and 4. Table I indicates that the Y and S matrices of a semireciprocal network have the same structure as the Z matrix. The systems coupling matrix of a semireciprocal distributed network on the other hand must have the form

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} & R_{14} \\ R_{21} & -R_{11} & R_{23} & R_{24} \\ R_{24} & -R_{14} & R_{33} & R_{34} \\ -R_{23} & R_{13} & R_{43} & -R_{33} \end{bmatrix}. \quad (16)$$

Compare this with the R matrix of a reciprocal network where the four elements of the lower left block must have signs opposite to that in (16).

A consistent set of conditions exists for a sixth property, namely, pure antireciprocity has also been established but left out of this discussion because we could find no physical example of a distributed four-port behaving like a generalized gyrator.

A computer program has been implemented that converts any listed complex 4×4 matrix into any other, and performs tests described in Table I as required.

V. APPLICATION

In anisotropic layered waveguides, generally the TE mode (line 1) is coupled to the TM mode (line 2). This transmission medium is noteworthy because it may or may not be bilaterally symmetric depending on the configuration and can be reciprocal or semireciprocal depending on the nature of the permittivity.

Solving the Maxwell equations in a stratified geometry depicted in Fig. 2, assuming a lossless, nonmagnetizable, homogeneous, and anisotropic dielectric, uniformity in the y (and z) direction and $\exp(j\omega t)$ time dependency, it can be shown [5] that (8) holds, where

$$a(x) = \Omega^{-1} \begin{bmatrix} \eta_0^{-1/2} E_y(x) \\ \eta_0^{1/2} H_z(x) \\ \eta_0^{-1/2} E_z(x) \\ -\eta_0^{1/2} H_y(x) \end{bmatrix} \quad (17)$$

and η_0 is the free-space wave impedance.

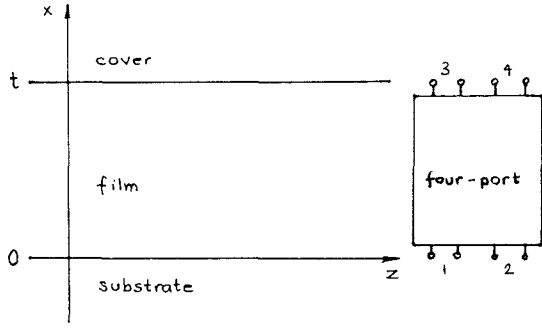


Fig. 2. Geometry of an anisotropic slab waveguide. All regions may be anisotropic. The electromagnetic wave propagates in the z direction. The four-port represents the film region.

For a uniaxial dielectric in polar configuration (optic axis in the y - z plane), or for a biaxial medium rotated around its crystalline x -axis, the relative permittivity matrix and the system coupling matrix is

$$[\epsilon_r] = \begin{bmatrix} \epsilon_{xx} & 0 & 0 \\ 0 & \epsilon_{yy} & \epsilon_{yz} \\ 0 & \epsilon_{yz}^* & \epsilon_{zz} \end{bmatrix} \quad (18)$$

and

$$R = \begin{bmatrix} \sqrt{\epsilon_{yy} - \beta^2} & 0 & 1/2\sqrt{Z_1 Z_2} \epsilon_{yz} & 1/2\sqrt{Z_1 Z_2} \epsilon_{yz} \\ 0 & -\sqrt{\epsilon_{yy} - \beta^2} & -1/2\sqrt{Z_1 Z_2} \epsilon_{yz} & -1/2\sqrt{Z_1 Z_2} \epsilon_{yz} \\ 1/2\sqrt{Z_1 Z_2} \epsilon_{yz}^* & 1/2\sqrt{Z_1 Z_2} \epsilon_{yz}^* & \sqrt{(1 - \beta^2/\epsilon_{xx})} \epsilon_{zz} & 0 \\ -1/2\sqrt{Z_1 Z_2} \epsilon_{yz}^* & -1/2\sqrt{Z_1 Z_2} \epsilon_{yz}^* & 0 & -\sqrt{(1 - \beta^2/\epsilon_{xx})} \epsilon_{zz} \end{bmatrix} \quad (19)$$

respectively, where the normalized line impedances are $Z_1 = [\epsilon_{yy} - \beta^2]^{-1/2}$ and $Z_2 = [(1 - \beta^2/\epsilon_{xx})/\epsilon_{zz}]^{1/2}$ and $\beta = k_z/k_0$ is the effective guide index. For a uniaxial dielectric in longitudinal configuration (optic axis in the x - y plane), or for a biaxial medium rotated around its crystalline z -axis, the relative permittivity matrix and the system coupling matrix is

$$[\epsilon_r] = \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & 0 \\ \epsilon_{xy}^* & \epsilon_{yy} & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \quad (20)$$

$$R = \begin{bmatrix} \sqrt{\frac{\Delta_{zz}}{\epsilon_{xx}} - \beta^2} & 0 & -\frac{\beta}{2} \sqrt{\frac{Z_1}{Z_2}} \frac{\epsilon_{xy}^*}{\epsilon_{xx}} & \frac{\beta}{2} \sqrt{\frac{Z_1}{Z_2}} \frac{\epsilon_{xy}^*}{\epsilon_{xx}} \\ 0 & -\sqrt{\frac{\Delta_{zz}}{\epsilon_{xx}} - \beta^2} & \frac{\beta}{2} \sqrt{\frac{Z_1}{Z_2}} \frac{\epsilon_{xy}^*}{\epsilon_{xx}} & -\frac{\beta}{2} \sqrt{\frac{Z_1}{Z_2}} \frac{\epsilon_{xy}^*}{\epsilon_{xx}} \\ -\frac{\beta}{2} \sqrt{\frac{Z_1}{Z_2}} \frac{\epsilon_{xy}}{\epsilon_{xx}} & -\frac{\beta}{2} \sqrt{\frac{Z_1}{Z_2}} \frac{\epsilon_{xy}}{\epsilon_{xx}} & \sqrt{\epsilon_{zz} \left(1 - \frac{\beta^2}{\epsilon_{xx}}\right)} & 0 \\ -\frac{\beta}{2} \sqrt{\frac{Z_1}{Z_2}} \frac{\epsilon_{xy}}{\epsilon_{xx}} & -\frac{\beta}{2} \sqrt{\frac{Z_1}{Z_2}} \frac{\epsilon_{xy}}{\epsilon_{xx}} & 0 & -\sqrt{\epsilon_{zz} \left(1 - \frac{\beta^2}{\epsilon_{xx}}\right)} \end{bmatrix} \quad (21)$$

respectively, where

$$Z_1 = [\Delta_{zz}/\epsilon_{xx} - \beta^2]^{-1/2},$$

$$Z_2 = [(1 - \beta^2/\epsilon_{xx})/\epsilon_{zz}]^{1/2},$$

and

$$\Delta_{zz} = \epsilon_{xx}\epsilon_{yy} - |\epsilon_{xy}|^2.$$

Inspection of the above coupling matrices, and the computed transfer matrices of an anisotropic layer corresponding to them, shows that the polar configuration is bilaterally symmetric, and if ϵ_{yz} is real (imaginary) then it is reciprocal (semireciprocal), whereas the longitudinal configuration is not bilaterally symmetric, and if ϵ_{xy} is real (imaginary) then it is semireciprocal (reciprocal). In addition, both configurations satisfy the losslessness condition and neither are transversally symmetric. Note also that there is no coupling between TE and TM modes in the longitudinal configuration when the direction of propagation is normal to the interface ($\beta = 0$), as in the case of liquid-crystal twist cells.

VI. CONCLUSION

Conservation laws have been presented in a form suitable to be applied to distributed, uniform integrated circuits. A new con-

cept, semireciprocity, has been introduced and a device configuration, to which it applies, demonstrated.

REFERENCES

- [1] H. J. Carlin and A. B. Giordano, *Network Theory* Englewood Cliffs, NJ: Prentice-Hall, 1964, ch. 4.
- [2] M. C. Pease, "Generalized coupled mode theory," *J. Appl. Phys.*, vol. 32, pp. 1736-1743, Sept. 1961.
- [3] G. Arfken, *Mathematical Methods for Physicists*. New York, NY: Academic Press, 1970, ch. 4.
- [4] M. C. Pease, "Conservation laws of uniform linear homogeneous systems," *J. Appl. Phys.*, vol. 31, pp. 1988-1996, Nov. 1960.
- [5] O. Schwelb and C. Baltassis, "Network theory for anisotropic stratified waveguides," presented at the 14th European Microwave Conference, Liège, Belgium, Sept. 10-13, 1984.